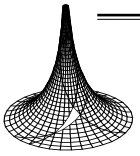

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tmcs@math.klte.hu
http://tmcs.math.klte.hu

Teaching
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Zbigniew Michalewicz – Matthew Michalewicz:

Puzzle Based Learning

An introduction to critical thinking, mathematics, and problem solving

Hybrid Publishers Melbourne 2008

Review by András Ambrus

Abstract. Based on their experiences with engineering, mathematics, computer science, business students concerning the puzzle based learning in different countries the authors summarize their main problem solving teaching ideas. With help of interesting, motivating, nice problems they analyze the main mathematical principles and problem types. The review gives an overview about the main ideas, results of an interesting book.

Key words and phrases: open ended problem, puzzle based learning, problem solving, critical thinking.

ZDM Subject Classification: D50, M10, M30, M40, U40.

Problem solving is no doubt an important part of mathematics education. Among the PISA competencies, *Problem posing and problem solving* is one of the eight main components. There are series of problem solving books published yearly around the world, which may raise the question: what can a new book in this domain offer us? In this review we will try to answer this question.

The first remark is that the authors are coming from Australia, from a quite different mathematics education culture. There is value in studying other viewpoints in this important educational domain.

The second remark is that the book devoted to university students – engineering, mathematics, computer science, business etc. – but can also be used in

secondary mathematics education. Another point is that mathematics books in higher education in Europe are mostly traditional (content – exercises).

Thirdly, the authors express quite openly that their book is based on the prior works of different experts in this domain: first of all M. Gardner and G. Polya, but they studied the work of other famous experts as well: Arthur Engel, Hugo Steinhaus, R. Smullyan. For us – for middle-European people – it is a pleasure of course to find among these experts after G. Polya others such as B. A Kordemski, D. O. Sklarjszkij, N. N. Csencov and I. J. Jaglom. These names prove that mathematics is important for the authors and that the puzzles are not only entertainment, but they have mathematical potential as well.

The book is based on a lot of experience, courses made in different countries, different universities (USA, Mexico, Argentina, New Zeeland, Australia, South Korea, Japan, China, Poland, Sweden, Germany, Spain, Italy, France, UK). This rich teaching experience is very important in our opinion because many times problem solving books only mirror the cleverness of the authors without any didactical experience or didactical remarks.

Puzzle – problem relation

The authors consequently use the term "puzzle" instead of the term "problem". In our view we can view the puzzles in the book as real problems.
Criteria for good educational puzzles are:

1. Generality: Educational puzzles should explain some universal mathematical problem solving principles. The general strategies would allow for solving new problems in the future. The authors are convinced that introducing and applying new strategies must be supported by the instructors.

2. Simplicity: Educational puzzles should be easy to state and easy to remember. We agree with the authors: *puzzles that are easy to remember increase the chance that the solution method will also be remembered.*

3. Eureka factor: A puzzle should be interesting because the result is counter-intuitive. A Eureka moment is reached when the correct path to solving the puzzle is recognized. Hence, the puzzles should have elementary solutions that are not obvious.

4. Entertainment factor: Educational puzzles should be entertaining; otherwise it is easy to lose interest in them. They can also be placed in an interesting setting, such as a casino, fight against dragons, dropping eggs from a tower, etc.

The characteristics of puzzle based learning:

– learning is driven by challenging, open-ended problems;
– students work in small cooperative groups;
– teacher takes the role of "facilitator" of learning.

Why is it important to use puzzles in education?

The puzzle-based learning approach may help us convince students that:

1. Science is useful and interesting.
2. The basic courses they are taking are relevant.
3. Mathematics is not that scary (there is no need to hate it!).
4. It is worthwhile to stay in school, get a degree and move into the real world which is loaded with interesting problems.

These points are important because most students are unclear about the significance of the topics covered during their studies. Very often they do not see a connection between topics taught and real-world problems, and, as a consequence, they lose interest.

The authors are of the opinion that the lack of problem solving skills in general is the consequence of decreasing level of mathematical sophistication in modern societies.

What is new in this book?

The authors compare their book with the traditional problem solving books (problem based, project based learning). In the latter versions, a major piece of work is conducted under the supervision of an experienced facilitator acting in a mentor role. The problems are quite complex, usually there is no single, clear, unique or correct way of proceeding.

In puzzle based learning, the problems are simpler and usually have a single, correct answer. The main aim of the authors is: *An important part of completing*

a puzzle is to understand what we have learned by solving the puzzle and how we can apply this knowledge to other problems. To express this idea in a modest way: the meta-cognitive aspects are important factors in problem solving teaching.

Some basic ideas concerning the problem solving teaching

G. Polya stated that if you want to learn to swim, you need to go into the water and to try some moves with your hand and legs in the water. The authors have similar opinion, related to Engel idea: problem solving can be learned only by solving problems. *But it must be supported by strategies provided by the trainer.* It is a clear opinion, opposite to other views, that the students can acquire the problem solving strategies by solving a lot of problems without extra analysis of the solution process and the used strategies. We must critically mention that the Middle-European style prefer the latter one.

Using unstructured problems

The idea is to increase the students' mathematical awareness and problem solving skills by discussing a variety of puzzles. It means that puzzle based learning focuses on getting students to think about framing and solving unstructured problems.
Puzzle based learning allows us to learn problem solving skills:

- by experience
- by imitation
- by reflection (Basic questions at this phase: *What are we learning? How are we learning? How are we using what we have learned?*

General rules for guiding the problem solving process

The authors formulated three rules which are fundamental during the problem solving process. Most of the problems discussed in the book are analyzed using these rules. This makes it possible for the reader to follow the basic steps of problem solving and hopefully repeat these steps in the future.

Rule 1: Be sure you understand the problem, and all the basic terms and expressions used to define it.

Before attempting to solve a problem, the solver needs to spend as much time as possible understanding all the aspects of the problem. The more time we spend on understanding (analysing) the problem, the less time it takes to find the solution! *Patience* is a prerequisite for effective problem solving. This phase is closely related to critical thinking, which refers to our ability to ask and answer critical questions related to what we have read or heard. The questions for critical thinking may be as follows: *Which words or phrases are ambiguous? What are the assumptions? What significant information is omitted?*

Rule 2: Do not rely on your intuition too much; solid calculations are far reliable.

Intuition is the result of a complex judgement based on experience – a judgement that probably can not be itemized or expressed in words. Intuition is often a bad advisor, first of all in probability. It is often mixed with emotions.

The question is not a simple one because very often good intuition leads to the main solution idea. Of course, the solver must control the idea given by intuition with mathematical, logical tools. The creative thinkers have good intuition.

Intuition is very private and the success rate is very questionable.

Rule 3: Solid calculations and reasoning are more meaningful when you build a model of the problem by defining its variables, constraints, and objectives.

The authors emphasize the distinction between the real-world problems and models. To build a model, approximations and simplifications are necessary.

"The assumed real world is abstracted from real situation by concentrating on the dominant variables that control the behaviour of the real system. The model, being an abstraction of the assumed real world, expresses in an amenable manner the mathematical function that represents the behaviour in the amended system. Finding a solution require some reasoning within our model"

The authors' experience is that these rules together with the entertaining puzzles which illustrate them would stay in the memory of the reader for a very long time.

The structure of the book

The book contains three parts and thirteen chapters.

In the first part the three rules are analysed by demonstrating concrete examples.

In the second part we find examples for mathematical principles and problem types. In these eight chapters the following questions are discussed and analyzed:

some mathematical principles, constraints, optimization, probability, statistically speaking, simulation, pattern recognition, and strategies.

The third part contains various puzzles with solutions and various assignments without solutions.

How to use this book?

Although the solutions of the problems – except the last chapter – are demonstrated in the book, the reader may try to solve the problems after reading the text of them. Of course this is the best, highest level. If the reader can solve some problems, it is worth studying the official solution too, to compare it with his (her) own one. I am sure that everybody can find good, creative solution ideas. Of course, for a lot of readers there are quite hard, complex problems with unusual and unknown solutions in the book. For these learners we can recommend Polya's statement: *The best students sometimes also need solution models, if they did not meet yet some original, creative ideas, methods used in the solutions process of a problem.*

To whom may we recommend this book?

We very warmly recommend this book for mathematics educators in universities and colleges. We are convinced that the secondary school mathematics teachers may profit a lot from it as well. Last but not least, this book is worthwhile to study for researchers dealing with problem solving teaching at all levels. Finally we may recommend for everybody who likes to solve puzzles.

Additional information on the book is available on the website: `www.PuzzleBasedLearning.edu.au`.

ANDRÁS AMBRUS
EÖTVÖS LÓRÁND UNIVERSITY
INSTITUTE OF MATHEMATICS
CENTER OF DIDACTICS OF MATHEMATICS
H-1117 BUDAPEST
PÁZMÁNY PÉTER SÉTÁNY 1/C
HUNGARY

E-mail: `ambrus@cs.elte.hu`

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